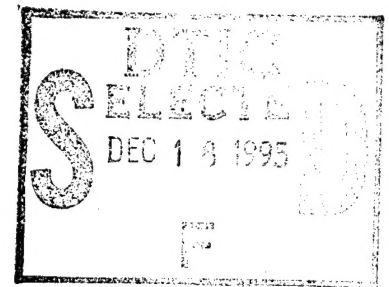NASA Technical Memorandum 80217

A PROBABILISTIC ANALYSIS OF ELECTRICAL

EQUIPMENT VULNERABILITY TO CARBON FIBERS
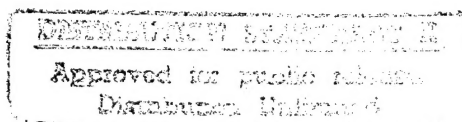
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SUMMARY

The statistical problems of airborne carbon fibers falling onto electrical circuits have been idealized and analyzed. The probability of making contact between randomly oriented finite-length fibers and sets of parallel conductors with various spacings and lengths has been developed theoretically. The probability of multiple fibers joining to bridge a single gap between conductors, or forming continuous networks has been included. From these theoretical considerations, practical statistical analyses to assess the likelihood of causing electrical malfunctions have been produced. These statistics have been confirmed by comparison with results of controlled experiments.

INTRODUCTION

Carbon fibers used in composite materials are electrically conductive. When such a composite material is burnt, the matrix material is usually softened or burnt first. The fibers can then be released as single fibers or bundles of fibers or even large structural fragments. An analysis of the risk posed by this release of fibers (ref. 1) has shown that the atmosphere can transport the single fibers for long distances, and that these fibers can cause malfunction or failure to some electrical equipment by producing short circuits across open terminals or circuits. Extensive experimentation was required to establish the vulnerability of equipment ranging from power insulators to aircraft transponders. The development of vulnerability models was necessary to simplify the testing, to allow extrapolation and to give confidence that the sample results were indicative of the behavior of classes of equipment.

The electronic circuit board shown in figure 1 is typical of those found in many electronic devices. It consists of components with bare leads and bare conductors on a flat non-conducting circuit board. The modeling approach taken in this

study was based on the assumption that any such circuit could be characterized by a number of conductor pairs of known spacings and known lengths, across which fibers might be deposited. The eight sketches of figure 2 represent idealized circuit configurations that have practical significance. The probability that randomly dropped fibers will cross the conductors as shown was solved analytically in this study for each of these configurations.

In figure 2(a), the basic idealization of a fiber falling on a pair of conductors in an indefinite array is shown. In figure 2(b), the configuration has been reduced to a fiber falling on only one pair of infinitely long conductors. In figure 2(c), the configuration is modified further to treat a fiber falling on a pair of conductors of finite length. In figure 2(d), a single conductor of finite length is crossed by a fiber. This configuration was analyzed as a step toward analysis of configuration in which more than one fiber joined to bridge one gap; the configuration also represents the case where two randomly dropped fibers cross each other. The configuration shown in figure 2(e) is of a very large number of fibers deposited on a surface and represents the logical limiting case in which any underlying circuitry would be compromised. Figures 2(f), 2(g), and 2(h) represent three possible configurations requiring two fibers to cause a failure. The circuit of figure 2(f) represents a situation in which a single fiber may not pass sufficient current to cause a circuit fault, so that two fibers in parallel are required to cause a failure. The circuit of figure 2(g) represents a case where two separate pairs of conductors must be bridged before the device malfunctions. In the circuit of figure 2(h), the conductive spacing exceeds the individual fiber length, so that only pairs of fibers can bridge the gap.

The analysis of each of the foregoing configurations is an extension of the classical solution by Buffon (ref. 2), who analyzed the probability of a randomly dropped needle intersecting any one of an infinite array of parallel lines. Because the present analyses build on the Buffon analysis, the Buffon analysis is summarized briefly.

The analysis developed in this study provided estimates of failure distributions for specified densities of fiber deposition for each of these configurations. Further, they provided estimates of the mean density of fiber deposition required to produce each of the configurations.

Experiments were conducted by dropping fibers on circuit boards with parallel conductors to check the analyses. The analyses, their predictions, experimental results and comparisons with predictions are presented in this report.

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LIST OF SYMBOLS

D	deposition density, f/m^2
D_f	deposition to failure, f/m^2
$\overline{D}$	mean deposition to failure, f/m^2
$\overline{D}_T$	mean deposition to failure due to pairs, f/m^2
$F(X/L)$	defined algebraic function
i	summing subscript
k	constant
L	fiber length, m
L_c	conductor length, m
Max( )	maximum value of enclosed variable
Min( )	minimum value of enclosed variable
n	number of intercepts with a given fiber
n_1	number of events per unit length
N_f	total number of fibers
$P[\ ]$	cumulative probability
P_f	probability of failure
$P(0), P(1), P(2)$	probability of intercepting ( ) conductors
$p(\ )$	probability density function
T	pair density, f/m^2
$\overline{T}$	mean pair density to failure, f/m^2
u	substitution variable
X	conductor spacing, m
x	fiber location coordinate, m
y	fiber location coordinate, m
θ	fiber orientation

$\theta_1, \theta_2, \phi_t, \phi_b$ specific orientation limits

ANALYSIS

A Fiber Lying Across One Line in an Infinite

Array of Parallel Lines

Figure 3 shows a fiber of length L lying across an infinite array of equally spaced parallel lines. The statistics of the intersection between the fiber and line in the array is known as Buffon's Needle Problem (ref. 2). For completeness, Buffon's solution of that problem is given here first.

If the location and orientation of the fibers are random, the probability of intersections with the array is insensitive to the y-position of the fibers, and is repetitive in the x-position. If the location of any fiber is defined by the coordinates of its left-hand end (arbitrary choice), then all possible fiber placements are reduced to locations $0 \leq x \leq X$ and orientations $-\pi/2 \leq \theta \leq \pi/2$.

If $L/X < 1$, no intersections are possible if $x < (X - L)$. For fibers with $x > (X - L)$, the limiting orientation θ_2 beyond which the fiber will not intersect the line is

$$\theta_1 = \cos^{-1}\left[\frac{X - x}{L}\right] \tag{1}$$

Because all orientations are equally likely, the probability that the fiber will intersect the line is the ratio $2\theta_1/\pi$.

For all possible x locations, the probability that the fiber will intersect one line is

$$P(1) = \frac{\displaystyle\int_0^{L-X} 0 \, dx + \int_{L-X}^{X} 2\cos^{-1}\left[\frac{X-x}{L}\right] dx}{\displaystyle\int_0^{X} \pi \, dx}$$

$$= \frac{2L}{\pi X} \int_0^1 \cos^{-1} u \, du = \frac{2L}{\pi X} \tag{2}$$

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Because a fiber cannot cross two lines, the probability that the fiber will not cross any line is the complement of $P(1)$ and is given by

$$P(0) = 1 - \frac{2L}{\pi X} .$$ (3)

A Fiber Lying Across Two Lines in an Infinite Array

of Parallel Lines

The extension of the Buffon needle problem to the intersection of a fiber with two lines provides the basis for electrical short-circuit analysis. If the fibers are in the range $1 \leq L/X \leq 2$, three results are possible. No lines are intersected, one line is intersected, or two lines are intersected.

In figure 4, a fiber originating at x will intersect no lines if $\theta > \theta_1$, it will intersect one line if $\theta_2 < \theta \leq \theta_1$, or it will intersect two lines if $\theta \leq \theta_2$, where

$$\theta_1 = \cos^{-1} \left[\frac{X - x}{L} \right]$$ (4)

and

$$\theta_2 = \cos^{-1} \left[\frac{2X - x}{L} \right]$$ (5)

The probability that the fiber will intersect two lines is

$$P(2) = \frac{\displaystyle\int_{2X-L}^{X} 2 \cos^{-1} \left[\frac{2X - x}{L} \right] dx}{\displaystyle\int_{0}^{X} \pi \, dx}$$

$$= \frac{2L}{\pi X} \left\{ \sqrt{1 - \left[\frac{X}{L} \right]^2} - \left[\frac{X}{L} \right] \cos^{-1} \left[\frac{X}{L} \right] \right\}$$ (6)

and the probability that no lines are intersected is

$$P(0) = \frac{\int_0^X \left\{ \pi - 2 \cos^{-1}\left[\frac{X-x}{L}\right] \right\} dx}{\int_0^X \pi\, dx}$$

$$= 1 + \frac{2L}{\pi X}\left\{ \sqrt{1 - \left[\frac{X}{L}\right]^2} - \left[\frac{X}{L}\right]\cos^{-1}\left[\frac{X}{L}\right] - 1 \right\} \tag{7}$$

And the probability that only one line is intersected is

$$P(1) = 1 - P(0) - P(2)$$

$$= \frac{2L}{\pi X}\left\{ 1 - 2\left\{ \sqrt{1 - \left[\frac{X}{L}\right]^2} - \left[\frac{X}{L}\right]\cos^{-1}\left[\frac{X}{L}\right] \right\} \right\} \tag{8}$$

In figure 5 these probabilities are plotted as a function of L/X. It shows that the probability that no lines are intersected decreases with fiber length, and that the probability of intersecting two lines increases rapidly with fiber length as the probability of intersecting none or only one line decreases.

A Fiber Lying Across Two Parallel Conductors

The simplest idealization of a vulnerable electrical circuit consists of two infinitely long parallel conductors which can be intersected by a fiber, as shown in figure 2(b). The statistics for this configuration are slightly different from those of the endless array. In the endless array, a fiber dropped anywhere had a probability of intersecting two lines definable in terms of L/X. With only two conductors in an infinite plane, the probability of intersecting the two conductors is meaningful for only those fibers deposited in a narrow strip surrounding the two conductors. A fiber of length L, whose left-hand end lies within a distance L-X to the left of the left conductor (fig. 6), has a probability of intersecting both conductors as follows:

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$$P(2) = \frac{\displaystyle\int_0^{L-X} \cos^{-1}\left[\frac{x + X}{L}\right] dx}{\displaystyle\int_0^{L-X} \pi/2 \; dx}$$

$$= \frac{2}{\pi\left(1 - \frac{X}{L}\right)} \left\{ \sqrt{1 - \left(\frac{X}{L}\right)^2} - \left(\frac{X}{L}\right) \cos^{-1}\left(\frac{X}{L}\right) \right\} \qquad (9)$$

This relationship is shown in figure 7. The probability of inter-secting both conductors is zero for fiber lengths less than the line spacing, and approaches $2/\pi$, or 0.6366, for very long fibers. This probability applies to only those fibers whose left-hand end lies in a strip of width $L-X$ to the left of the conductor pair, and hence includes all fibers which can cross both conductors, and none that cannot possibly reach across the pair.

A Fiber Lying Across Two Parallel Conductors

of Finite Length

Figure 8 shows two parallel conductors of length L_c crossed by a fiber whose left end is at x,y. For each fiber location x,y, the range of orientations for which both conductors will be intersected is limited by both the length of the fibers and the length of the conductors. As shown in figure 8, the fiber will fail to cross the axis of the conductor on the right if the orientation angle exceeds

$$\theta_t = \cos^{-1}\left(\frac{x + X}{L}\right)$$

or the fiber will miss the top end of the conductor on the right if

$$\phi_t = \tan^{-1}\left(\frac{\frac{L_c}{2} - y}{X + x}\right)$$

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Similarly at the bottom end

$$\theta_b = -\cos^{-1}\left(\frac{x + X}{L}\right)$$

$$\phi_b = -\tan^{-1}\left(\frac{\frac{L_c}{2} + y}{X + x}\right)$$

At the point x, y the probability of intersecting both conductors is given by

$$P(2) = \frac{\text{Min}(\phi_t,\ \theta_t) - \text{Max}(\phi_b,\ \theta_b)}{\pi}$$

The closed form integration for all viable values of x and y is not believed to be possible, but a numerical integration was performed for various ratios L_c/L, and various ratios x/L.

Double intercepts are possible for only as many fibers as have their left ends in the rectangle $L_c(L-X)$ to the left of the left conductor. Not all fibers satisfying that criterion participate, but an equal number of symmetrically placed fibers lying outside the rectangle do participate; thus, the total number is appropriate. Normalizing the answer by this number also provided a non-dimensional presentation of results. The numerical results are presented in table I and are plotted in figure 9. The plot shows that short conductors have a lower intercept probability than long conductors, but that the difference vanishes for close spacing of conductors $(X/L = 0)$. The probability is essentially unaffected by conductor length if $L_c > 10L$.

For practical applications of this analysis, the foregoing result is usually desired in terms of the number of "failures" for a given density of fiber deposition or the density of fiber deposition required to produce a failure. Only the mean values of these quantities are developed herein.

If the deposition density in the plane is D, the expected number of double intersections per unit length of conductor is

$$n = D(L - X)P(2)$$

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and for conductors at length L_c

$$n = DL_c(L - X)P(2)$$

$$= \frac{2}{\pi} DLL_c \left\{ \sqrt{1 - \left(\frac{X}{L}\right)^2} - \frac{X}{L} \cos^{-1}\left(\frac{X}{L}\right) \right\} \qquad (10)$$

$$= kD$$

Actually, the number of intercepts is a random variable with a Poisson distribution whose mean is the value just presented. The corresponding expected value of deposition density is then

$$\bar{D} = \frac{1}{k} = \frac{\pi}{2LL_c \left\{ \sqrt{1 - \left(\frac{X}{L}\right)^2} - \frac{X}{L} \cos^{-1}\left(\frac{X}{L}\right) \right\}} \qquad (11)$$

Most practical problems involve a variety of conductor spacings and lengths. The number of intercepts for such cases may be estimated by superposition unless fiber pairs join to bridge conductor gaps in significant numbers. The summation has the form

$$n = \frac{2}{\pi} DL \sum_{i=1}^{m} L_{ci} F\left(\frac{X_i}{L}\right) \qquad (12)$$

Fiber Lying Across a Single Conductor of Finite Length

The probability of a fiber intersecting a single conductor of finite length (Fig. 2d) is simply the limiting case of the solution from the previous section (when $\frac{X}{L} = 0$), or $\frac{2}{\pi}$. The corresponding number of intercepts is

$$n = \frac{2}{\pi} DLL_c \qquad (13)$$

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Fiber Pair Formation

The probability that two fibers will cross each other is a special case of a fiber falling on a single conductor when $L = L_c$. However, the total length of fibers per m^2 that are eligible to be intersected by a given fiber is DL. Consequently, the number of such intersections with one fiber for a given deposition density is

$$n = \frac{2}{\pi} DLL_c \qquad \text{from Equation} \quad (13)$$

and the total number of such intersections per unit area is

$$n = \frac{2}{\pi} D^2 L^2 \qquad (14)$$

Because each intersection involves two fibers, the total number of pairs per m^2, T, generated is $n/2$ and, provided $n << D$.

$$T = \frac{1}{\pi} D^2 L^2 \qquad (15)$$

The corresponding mean deposition density required to produce a pair is

$$D = \frac{\sqrt{\pi}}{L} \qquad (16)$$

Fiber Network Formation

The formation of a continuous network of fibers, as shown in Figure 2e, requires that each fiber be intersected by two or more other fibers. Although this is a necessary, it is not a sufficient criterion. However, it leads to a mean deposition that is lower than is actually required and simplifies the analysis. If $n = 2$ in Equation 14

$$2 = \frac{2}{\pi} D L^2$$

and

$$D = \frac{\pi}{L^2} \qquad\qquad (17)$$

A somewhat higher deposition is required to assure a complete network.

The Multifiber Failure Model

Figures 2(f), 2(g), and 2(h) show three schematic circuit traces with potential multifiber failure modes. For the first two of these modes the probability of interception is the same as that for single fibers. Only the statistics for the deposition density required to cause a failure differ from those for the single-fiber case.

Two fibers crossing a conductor pair. For the failure mode requiring two fibers on a single circuit as shown in figure 2(f), the probability distribution for the second intercept can be shown to be

$$P_f = 1 - (1 + D/\bar{D}) \; e^{-D/\bar{D}}$$

for which the probability density function is

$$p_f = D/\bar{D} e^{-D/\bar{D}}$$

and the expected value

$$\bar{D}_2 = \int_0^\infty D p_f dD = 2\bar{D} \qquad\qquad (18)$$

This probability of failure behaves approximately like D^2, and the mean deposition to failure is twice the mean deposition for a failure requiring only one fiber.

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Two fibers crossing two different conductor pairs. For the failure mode requiring one fiber on each of two circuits (Fig. 2(g)), the probability of failure is the product of the individual probabilities of failure or

$$P_f = (1 - e^{-D/\bar{D}})^2$$

for which the probability density function is

$$p_f = 2(e^{-D/\bar{D}} - e^{-2D/\bar{D}})$$

and the expected value is

$$D_2 = \int_0^\infty D \ p_f \ dD = \frac{3\bar{D}}{2} \tag{19}$$

For this mode the probability of failure also behaves approximately like D^2 but the mean deposition to failure is only one and one-half times the mean deposition to failure for the individual circuits.

Two fibers join to cross one pair of conductors. For the failure mode in figure 2(h), caused by fiber pairs, the calculation of the interception probability is significantly more complex than for single fibers. The geometric parameters for the arrangement of a pair of single fibers, shown in figure 10, have many degrees of freedom. To simplify the problem all pairs were assumed to link colinearly without overlap. Thus the length of paired fibers equals twice the individual fiber length.

Using the double length the expected number of intercepts is

$$n = \frac{4}{\pi} \ T \ L \ L_c \ F(\frac{X}{2L}) \tag{20}$$

For circuit spacings $L < X < 2L$ the mean pair deposition density to intercept two conductors is

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$$\bar{T} = \frac{\pi}{4L \ L_C \ F(\frac{X}{2L})} \tag{21}$$

The failure probability is exponential in terms of pair deposition

$$P_f = 1 - e^{-T/\bar{T}}$$

In Eq. (15), the pair deposition density was found to be

$$T = \frac{1}{\pi} D^2 L^2$$

so that the probability of failure in terms of single fiber deposition is

$$P[D_f \leq D] = 1 - e^{-D^2 L^2 / \pi \ \bar{T}}$$

or

$$= 1 - e^{-\frac{4D^2}{\pi \bar{D}_T}}$$

where the mean single fiber deposition density $\bar{D}_T$ at which failure due to pairs occurs is

$$\bar{D}_T = \frac{\pi}{2L} \sqrt{\bar{T}}$$

$$= \sqrt{\frac{\pi^3}{16 \ L^3 \ L_C \ F(X/2L)}} \tag{22}$$

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EXPERIMENTAL EVALUATION

Tests were conducted to verify the predicted fiber depositions required to cause circuit failures. Two circuit boards were built with sets of exposed conductors of 100 mm length and the spacings shown in the following table. One board contained the first six conductor pairs, the second board contained the conductor pairs 7 and 8.

Conductor Pair	Spacing, mm
1	1.67
2	2.5
3	5.0
4	10.0
5	20.0
6	25.0
7	50.0
8	100.0

These circuit boards were exposed to a deposition of 5 mm long carbon fibers in the Ballistics Research Laboratory Test Chamber. The test apparatus and procedures are described in reference 3. Each circuit board was exposed until a short circuit developed, and the test was repeated 12 times.

Table II gives the deposition density on the circuit board at the time of failure. Figure 11 shows the mean deposition density to failure as a function of the fiber-length-to-gap ratio. Also plotted are the appropriate model predictions for single fiber failures, the pair failures, and the criterion for network formation.

The agreement between these experimental data and the model predictions was regarded adequate for the carbon fiber vulnerability assessment. Figure 12(a) is a cumulative probability plot of the failure data for the test series with $\frac{X}{L} = \frac{1}{3}$. The dominant failure mode is expected to be the single fiber mode. Consequently, the statistical distribution of the data is expected to be exponential distribution and represents a good fit to the data.

Figure 12(b) is a cumulative probability plot of the test series with $\frac{X}{L} = 2$. The dominant failure mode is multiple fiber mode such as derived for the deposition of fiber pairs. The solid curve in figure 14 represents the theoretical distribution and fits the data well.

CONCLUDING REMARKS

The probability of failure in electrical circuits exposed to airborne carbon fibers has been modeled for the case of plane deposition. Such probabilistic models were derived for single and multiple fiber problems.

The results of the analysis are summarized in Table III for the convenience of the reader.

Data from tests in which simple electrical circuits were exposed to carbon fibers show that the models provide good estimates of both the means and the distributions for the deposition to failure.

REFERENCES

1. Anon.: Assessment of Carbon Fiber Electrical Effects. NASA CP 2119, March 1980.

2. Feller, W.: An Introduction to Probability Theory and Its Application, Vol. II, 2nd ed., Wiley, New York.

3. Smith, Jill H., and Morrissey, John A.: Selection and Validation of a Multiple Fiber Model. US Army Ballistic Research Laboratory, ABRL-TR-02219, March 1980. (Available as NASA CR-159343.)

TABLE I.- INTERCEPT PROBABILITIES FOR FINITE LENGTH CONDUCTOR PAIRS

L_c/L \ X/L	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.636	0.529	0.471	0.421	0.376	0.335	0.297	0.259	0.216	0.161
2	.636	.564	.516	.417	.428	.386	.342	.296	.243	.176
3	.636	.476	.532	.488	.447	.402	.358	.309	.253	.181
4	.636	.582	.539	.496	.454	.411	.365	.315	.257	.183
5	.636	.585	.543	.502	.459	.416	.370	.319	.260	.185
6	.636	.588	.547	.505	.463	.419	.373	.321	.262	.186
7	.636	.589	.549	.508	.465	.421	.375	.323	.263	.186
8	.636	.591	.550	.509	.467	.423	.376	.324	.264	.187
9	.636	.592	.551	.511	.469	.424	.378	.325	.265	.187
10	.636	.593	.553	.512	.470	.426	.379	.326	.265	.188
∞	.636	.600	.562	.522	.480	.436	.388	.334	.271	.191

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TABLE II.- DEPOSITION DENSITIES OBSERVED IN TESTS(a)

X/L Run	1/3	1/2	1	2	4	5	10	20
1	2.4 E3	8.7 E4	8.4 E4	3.2 E4	1.2 E5	1.4 E5	1.3 E5	1.3 E5
2	7.4 E3	5.1 E3	8.6 E3	1.6 E5	3.5 E4	1.5 E5	1.4 E5	1.2 E5
3	7.3 E3	3.6 E4	1.9 E4	3.4 E4	9.7 E4	1.9 E5	1.3 E5	1.2 E5
4	3.4 E3	3.4 E3	2.0 E4	8.2 E4	1.9 E5	1.9 E5	1.3 E5	1.4 E5
5	1.2 E4	1.2 E4	3.2 E4	1.0 E5	8.8 E4	1.1 E5	1.1 E5	1.1 E5
6	1.2 E3	1.2 E3	1.6 E4	7.0 E4	1.1 E5	1.7 E5	1.4 E5	1.2 E5
7	1.4 E4	1.6 E3	5.1 E3	9.5 E4	1.2 E5	2.2 E5	1.8 E5	1.6 E5
8	2.3 E5	8.3 E3	2.9 E4	1.1 E5	1.3 E5	2.0 E5	1.8 E5	1.5 E5
9	8.2 E3	9.0 E3	1.6 E5	1.6 E5	1.2 E5	1.8 E5	1.2 E5	1.1 E5
10	1.3 E3	4.0 E3	1.2 E5	6.8 E4	1.3 E5	1.8 E5	1.8 E5	1.5 E5
11	2.0 E3	1.7 E4	6.0 E4	3.6 E4	9.7 E4	1.9 E5	1.2 E5	1.2 E5
12	8.0 E3	3.7 E4	2.5 E4	5.3 E4	6.8 E4	1.8 E5	1.9 E5	1.8 E5
Mean	7.52 E3	1.85 E4	3.62 E4	8.33 E4	1.09 E5	1.75 E5	1.45 E5	1.34 E5

(a) Each value represents deposition density (in fibers/m^2) at "failure" for fibers 5 mm long dropped on a circuit board having nine 100-mm-long conductors spaced as indicated.

TABLE III.- SUMMARY OF ANALYTICAL RESULTS

Configuration	Probability of Intersection	Number of Intersections for Given Deposition	Mean Deposition to Satisfy Configuration	Remarks
(diagram)	$\dfrac{2L}{\pi X}$	$\to \infty$	$\to 0$	Buffon's Solution Applies for $L < X$
(diagram)	$\dfrac{2L}{\pi X}\sqrt{1-\left(\dfrac{X}{L}\right)^2} - \dfrac{X}{L}\cos^{-1}\left(\dfrac{X}{L}\right)$	$\to \infty$	$\to 0$	Applies for $X < L < 2X$
(diagram)	$\dfrac{2}{\pi\left(1-\dfrac{X}{L}\right)}\left\{\sqrt{1-\left(\dfrac{X}{L}\right)^2} - \dfrac{X}{L}\cos^{-1}\left(\dfrac{X}{L}\right)\right\}$	$\dfrac{D}{L - X}$	$\dfrac{1}{k}$ (a)	Applies per unit length of conductor
(diagram)	See Figure 9	—	$\dfrac{1}{k}$ (a)	—
(diagram)	$\dfrac{2}{\pi}$	$\dfrac{2}{\pi}DLL_c$	—	—
(diagram)	$\dfrac{2}{\pi}$	$\dfrac{1}{\pi}DL^2$	N.A.	—
(diagram)	N.A.	N.A.	$\dfrac{L^2}{\pi}$	—
(diagram)	$\dfrac{2}{\pi\left(1-\dfrac{X}{L}\right)}\left\{\sqrt{1-\left(\dfrac{X}{L}\right)^2} - \dfrac{X}{L}\cos^{-1}\left(\dfrac{X}{L}\right)\right\}$	N.A.	$\dfrac{3}{2k}$ (a)	Applies per unit length of Conductor
(diagram)	$\dfrac{2}{\pi\left(1-\dfrac{X}{L}\right)}\left\{\sqrt{1-\left(\dfrac{X}{L}\right)^2} - \dfrac{X}{L}\cos^{-1}\left(\dfrac{X}{L}\right)\right\}$	N.A.	$\dfrac{2}{k}$ (a)	Applies per unit length of conductor
(diagram)	$\dfrac{2}{\pi\left(1-\dfrac{X}{2L}\right)}\left\{\sqrt{1-\left(\dfrac{X}{2L}\right)^2} - \dfrac{X}{2L}\cos^{-1}\left(\dfrac{X}{2L}\right)\right\}$	N.A.	$\dfrac{\pi^3}{16\,L^3 L_c F\left(\dfrac{X}{2L}\right)}$ (b)	—

(a) $k = \dfrac{2LL_c}{\pi}$

(b) $F\left(\dfrac{X}{2L}\right) = \sqrt{1 - \left(\dfrac{X}{2L}\right)^2} - \dfrac{X}{2L}\cos^{-1}\left(\dfrac{X}{2L}\right)$

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Figure 1.- Typical circuit board.
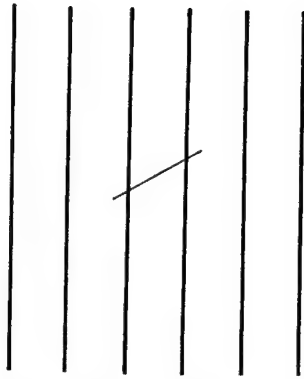
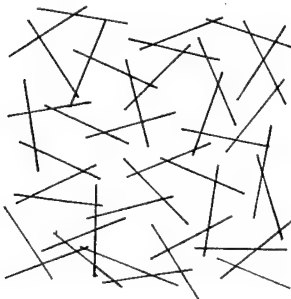a) Fiber lying across two in an infinite array of parallel conductors.

b) Fiber lying across a pair of infinitely long conductors.

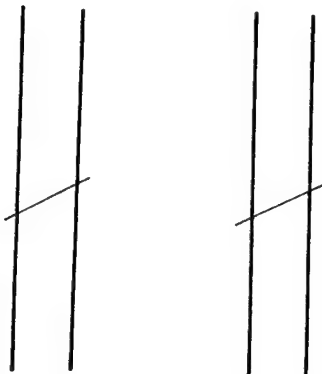c) Fiber lying across a pair of finite-length conductors.

d) Fiber lying across a single conductor of finite length or another fiber.

e) Fibers forming a continuous network.

f) Two fibers crossing a single pair of conductors.

g) Two fibers crossing two different pairs of conductors.

h) A pair of fibers crossing a single pair of conductors.

Figure 2.- Fiber-Conductor Configurations Analyzed.
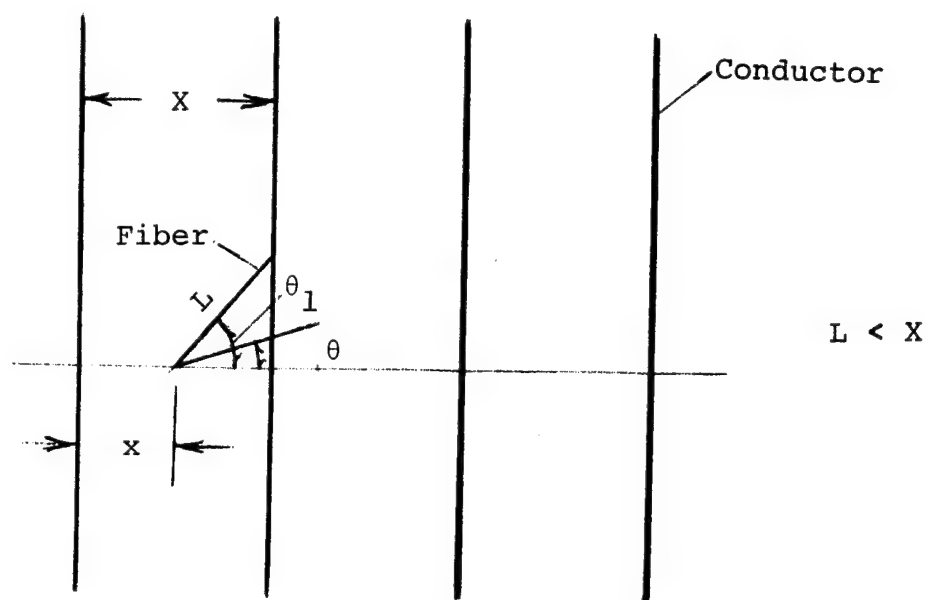
Figure 3.- Single fiber lying across one in an infinite
array of conductors.
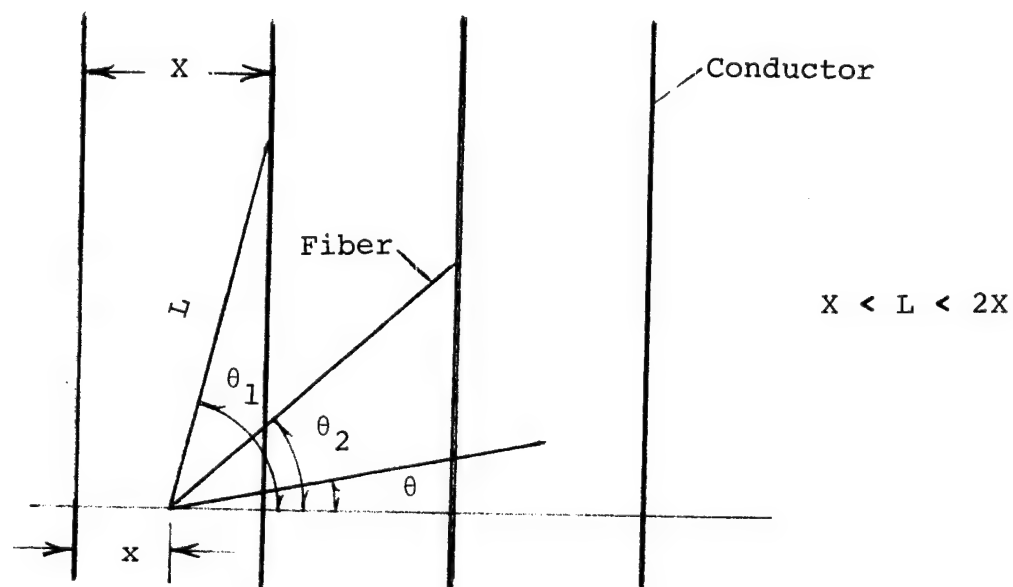
Figure 4.- Single fiber lying across two in an infinite
array of conductors.
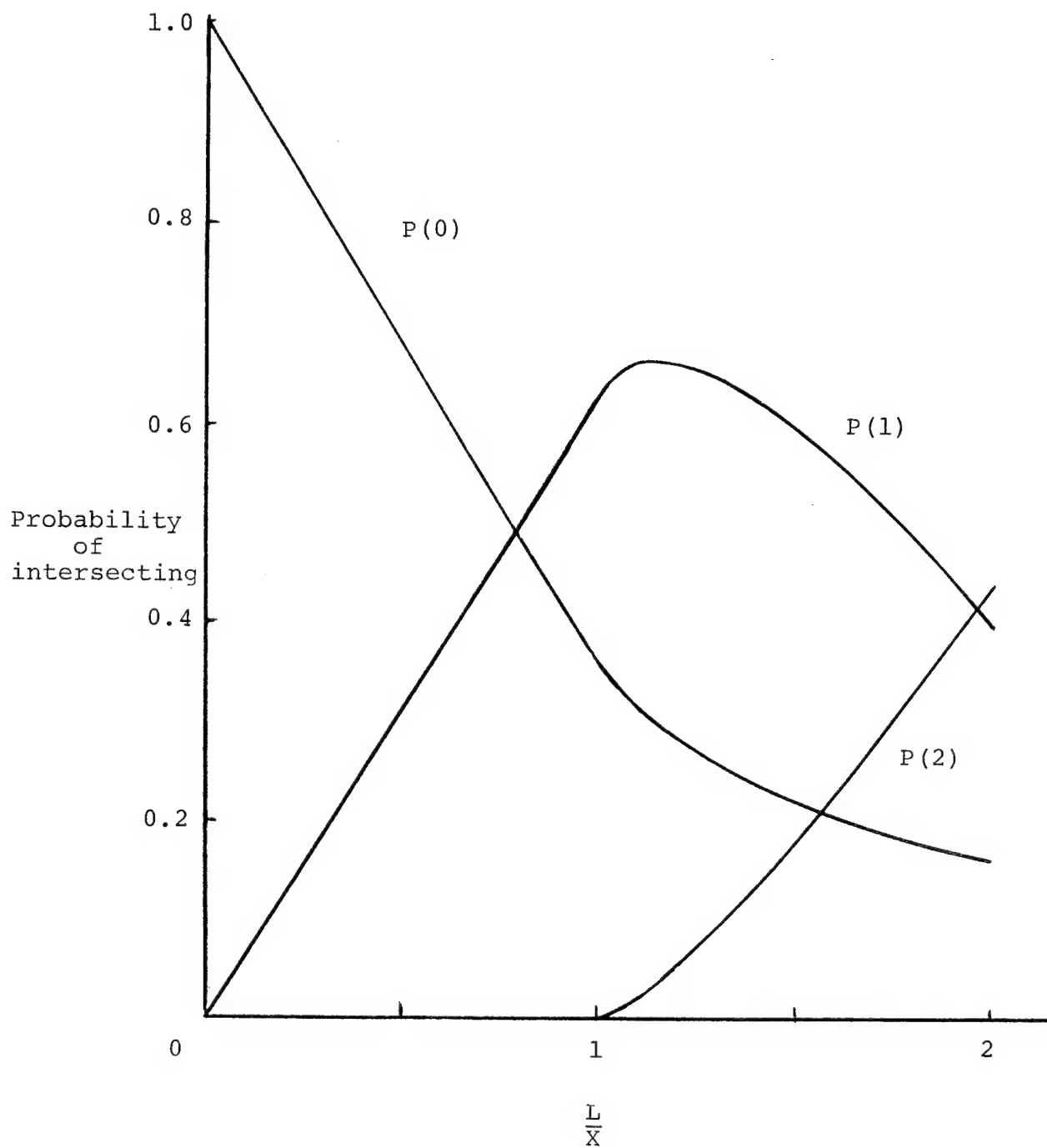
Figure 5.- Probability of a fiber intersecting none, one, or two of an infinite array of conductors.
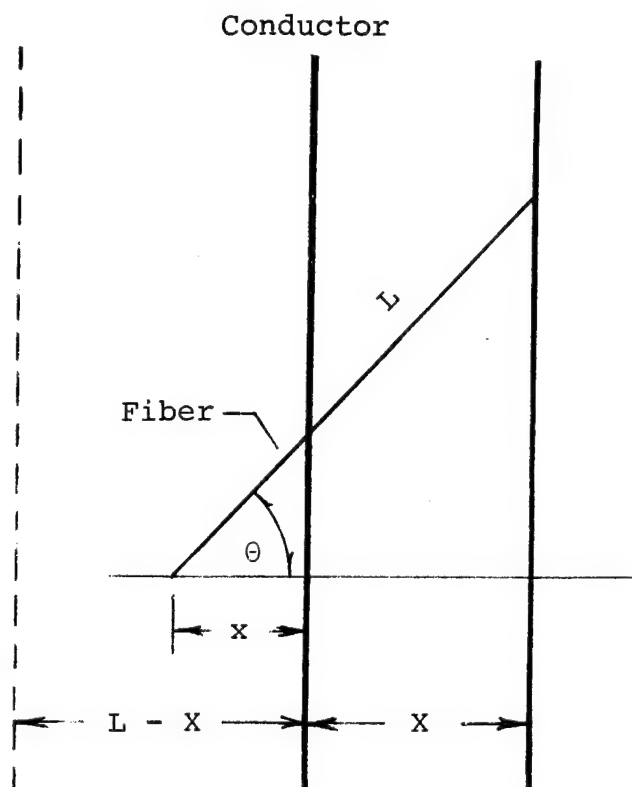
Figure 6.- Single fiber lying across two infinitely long
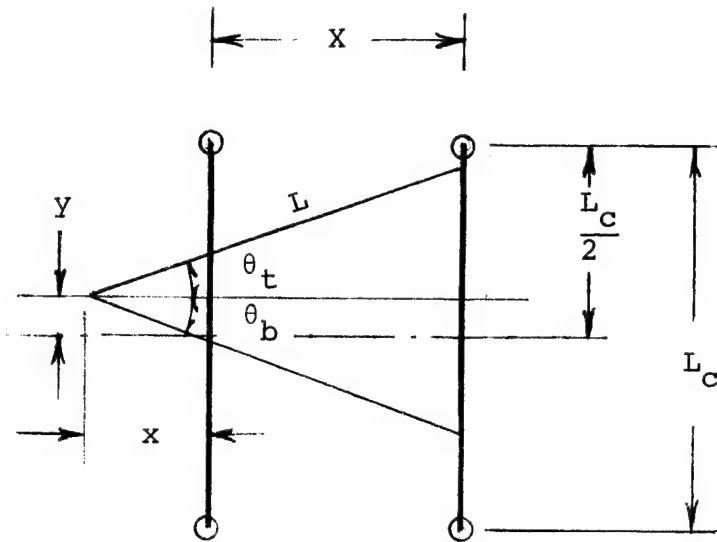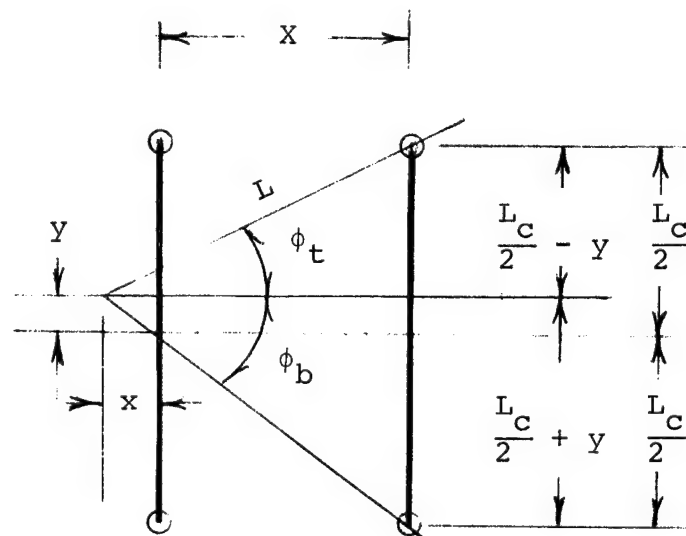conductors.
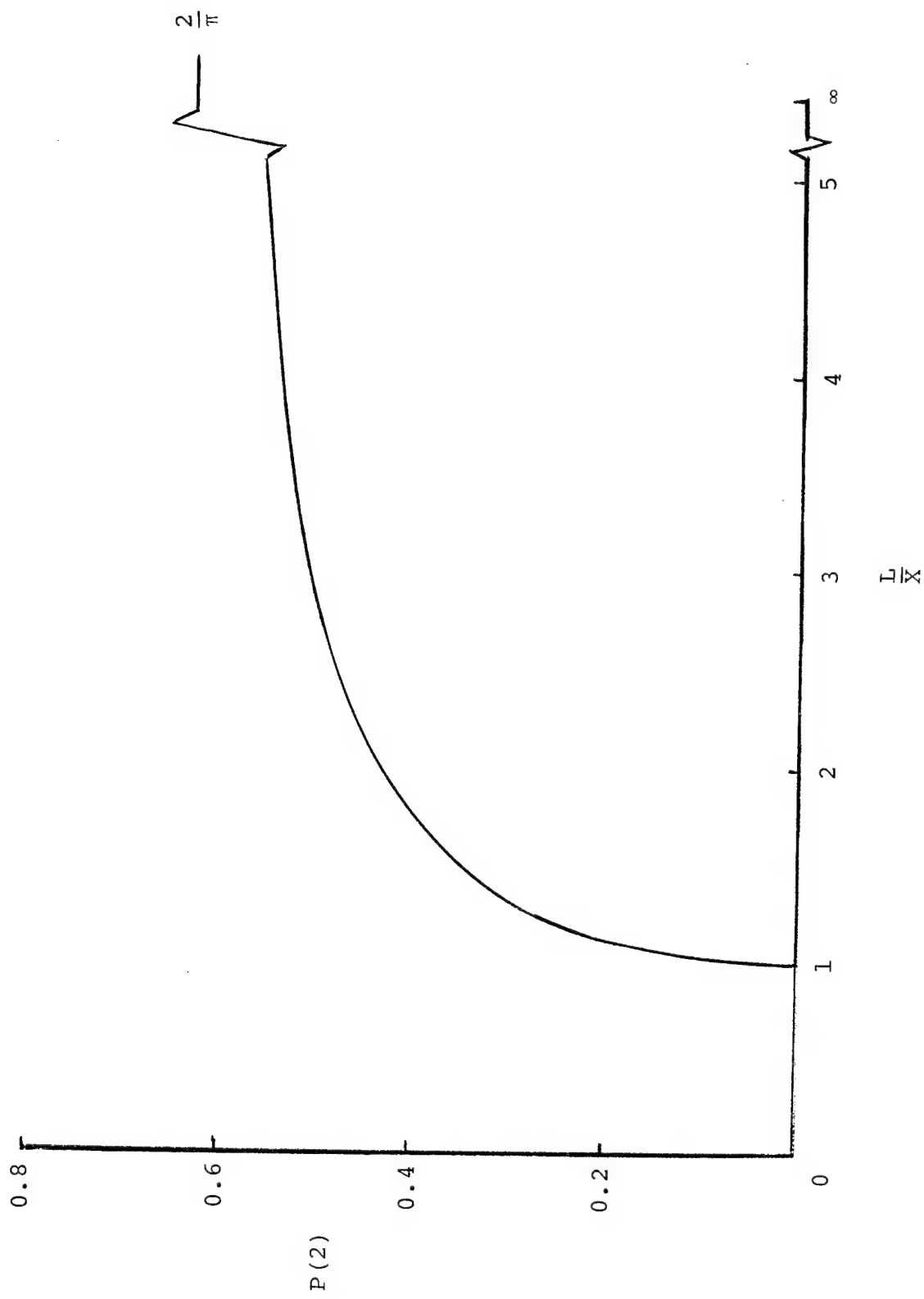
Figure 7.- Probability of one fiber intersecting both of two infinitely long conductors.

(a) Criterion for missing axis of one conductor.

(b) Criteria for missing ends of one conductor.

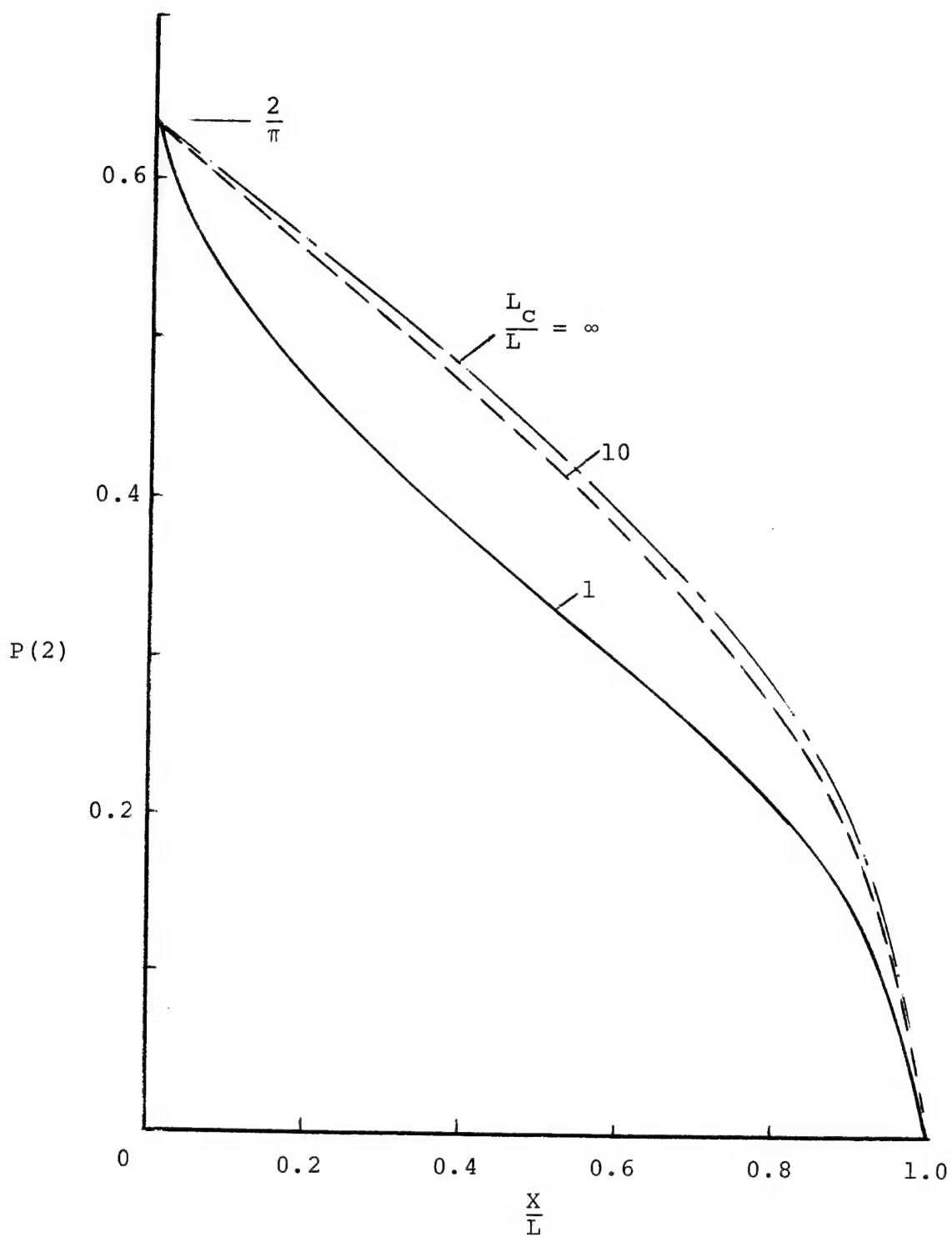Figure 8.- Single fiber lying across two finite-length conductors.

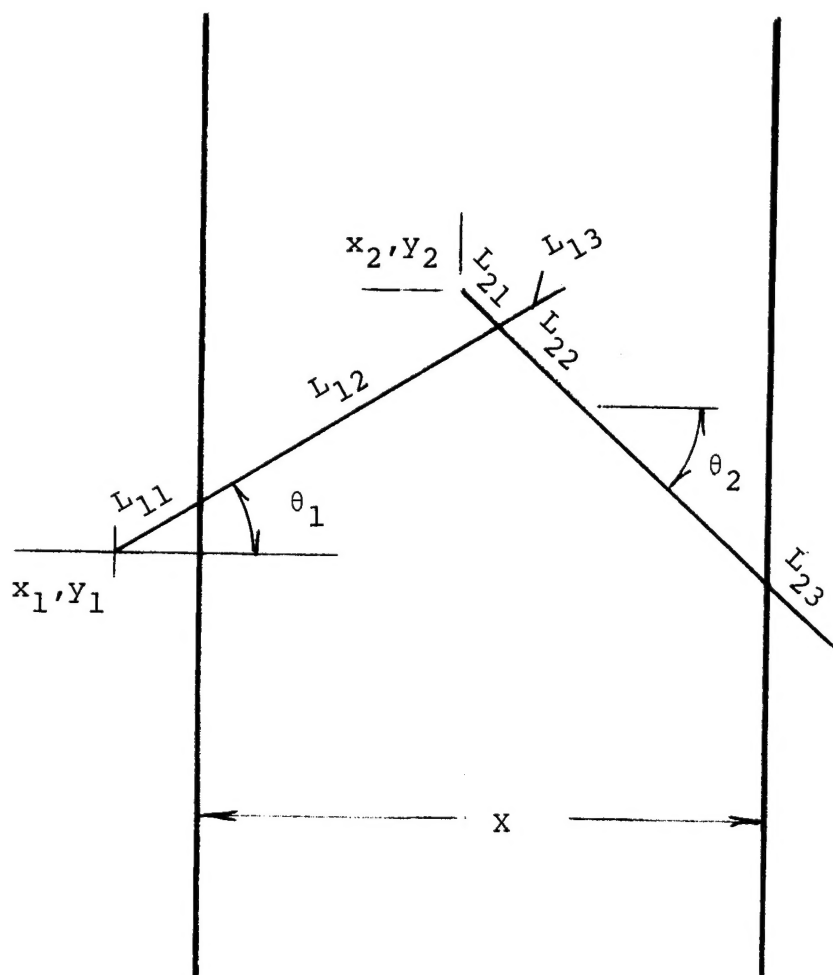Figure 9.- Probability of one fiber crossing both of two finite-length conductors.
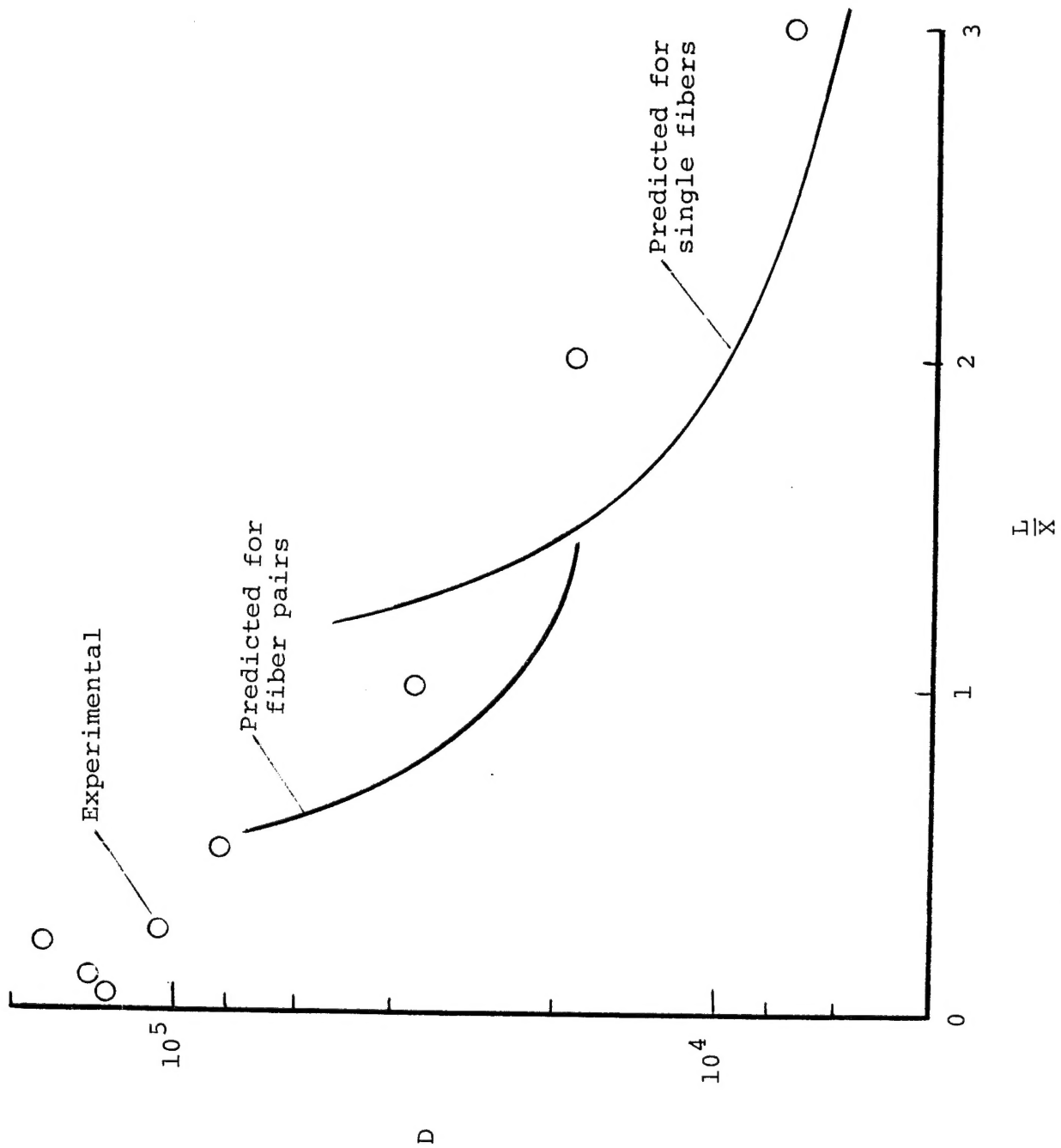
Figure 10.- Formation of fiber pairs.

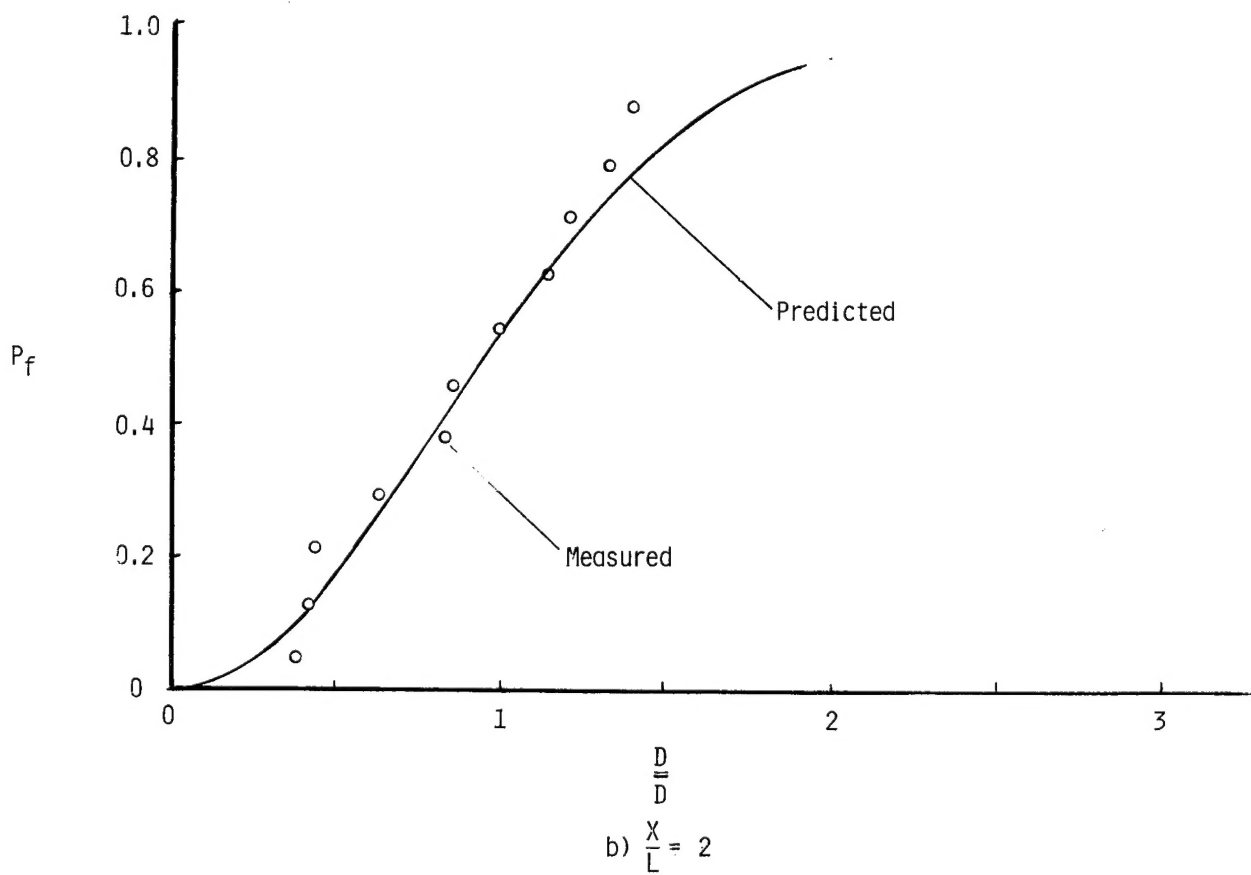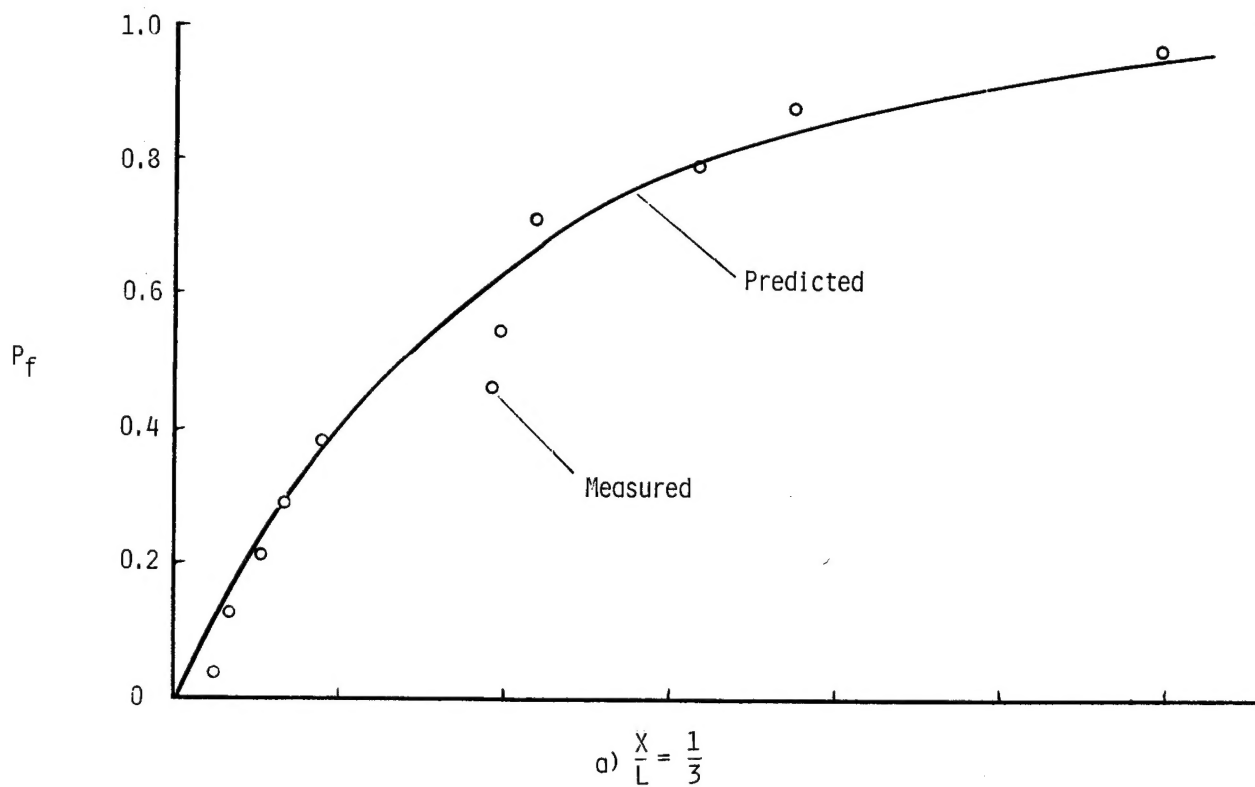Figure 11.- Deposition densities required to cause failures.

a) $\frac{X}{L} = \frac{1}{3}$

b) $\frac{X}{L} = 2$

Figure 12.- Experimental and Predicted Probabilities of Failure.

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15. Supplementary Notes

16. Abstract

  The statistical problems of airborne carbon fibers falling onto electrical circuits have been idealized and analyzed. The probability of making contact between randomly oriented finite-length fibers and sets of parallel conductors with various spacings and lengths has been developed theoretically. The probability of multiple fibers joining to bridge a single gap between conductors, or forming continuous networks has been included. From these theoretical considerations, practical statistical analyses to assess the likelihood of causing electrical malfunctions have been produced. These statistics have been confirmed by comparison with results of controlled experiments.

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